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'A HISTORICAL ANGLE', A SURVEY OF RECENT LITERATURE ON THE USE AND VALUE OF HISTORY IN GEOMETRICAL EDUCATION

ABSTRACT. In this article we give a survey of recent literature on the use and value of the history of geometry in mathematics education. It turns out that many authors have contributed to the debate why we should apply history. Their arguments can be divided into conceptual, (multi-) cultural and motivational ones. The number of authors who concentrated on the methodological question "How could we introduce history into mathematics lessons?" is considerably smaller. Those who want to make history an integral part of mathematics education have to bridge the gap between theoretical arguments and practical ideas, and, connected with this, between historians and teachers, and ultimately between mathematicians from the past and present day students. We divided the various surveyed publications, mainly journal articles, into categories according to a framework in order to clarify the discussion on the role of the history of geometry in education. Details about content and purpose of the articles are presented in two appendices, which make the articles more accessible both for further research and for practical teacher purposes.

KEY WORDS: education, geometry, history, mathematics, survey

1. INTRODUCTION

1.1. *Problem setting*

This article provides a systematic survey which lists the recent literature on the use and value of history in mathematics education, as far as the teaching of geometry is concerned. In recent years many authors have described why and how they use history of mathematics in teaching. This article brings order into the various incidental articles. Most publications are anecdotic and tell the story of one specific teacher, whereas it is unclear whether and how the (generally positive) experiences can be transferred to other teachers, classes and types of schools.

1.2. *Background*

In the last decade many researchers investigated the use and value of history in mathematics education which culminated in the ICMI-study (ICMI: International Commission on Mathematics Instruction) about the role of

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the history of mathematics in the teaching and learning of mathematics. The study started in 1997 with a discussion document by Fauvel and Van Maanen (1997) and resulted in the comprehensive report *History in mathematics education: the ICMI study*, Fauvel/Van Maanen (2000).

Another illustration to underline the current relevance of the subject is the growing interest among teachers in the history of mathematics. This is evident from the results of two questionnaire surveys. Fraser and Koop (1978) and Philippou and Christou (1998) reveal that teachers are interested in the history of mathematics, but, at the same time, are not well resourced to actually use such material in their own teaching. A pilot study carried out in 41 secondary schools in Hong Kong presents similar results in Lit et al. (1999).

Finally, major evidence for the genuine interest in the application of history to mathematics education can be found in the number of research groups in this field. For example, in Italy there are at least five active groups. In Denmark the history of mathematics is part of the official secondary school curriculum. In France we see intensive work in this area done by the IREMs (IREM: Institut de Recherche sur l'Enseignement des Mathématiques).

The discussion is of long standing. However, the innovative turn in recent years is the search for a theoretical underpinning and a methodology for the integration of history in the teaching and learning of mathematics.

In September 1998 our research group at the University of Groningen, The Netherlands, began a study into the use and value of history in teaching geometry. This project, entitled 'Reinvention of Geometry', focuses on converting the results of studies on the history of geometry into resource material appropriate for geometry lessons at school. The influence of the material produced on the quality of teaching and learning will be evaluated.

1.3. *Research methodology*

This article presents a survey of literature that has become available since 1970, with an emphasis on the nineties. The initial year was chosen because of the end of the New Math movement in which the content of mathematical education was fundamentally reconsidered. The survey is based on articles in the English, German and French languages in a number of leading journals[1] for mathematical educational research. Also, Dutch and Belgian journals are included, because the subject is actively researched in the Dutch language domain, whereas the respective publications are not readily accessible to the international audience.

The surveyed articles have been traced by roughly leafing through a fairly long series of subsequent volumes. With the help of the Eric database a list of supplementary articles on the combination of the three items: geometry-history-education, could be composed. More titles were provided by Smid (1998), who produced a bibliography of publications in the Dutch and German languages on the history of mathematics in relation to education. In his search for articles he surveyed the *Zentralblatt für Didaktik der Mathematik* completely from 1969 until 1998. Fauvel published a list of relevant articles in the *Newsletter* of the British Society for the History of Mathematics, which have been used as far as available. We also considered the bibliography at the end of the book *History in mathematics education: the ICMI study*, Fauvel/Van Maanen (2000). Finally there are some incidental articles from other journals.

2. *Why* USE HISTORY

The question *why* we could, or even should, use the history of mathematics in teaching and learning mathematics has been answered in a variety of ways in the articles under survey. Fauvel (1991) for example sums up about twenty reasons for applying the history to teaching and learning mathematics. To get an overview of the arguments mentioned by Fauvel and others we present a framework for bringing some structure into the variety of arguments.

2.1. *Conceptual arguments*

2.1.1. *Relevant to teachers*

"Effective learning requires that each learner has to retrace the main steps in the historical evolution of the studied subject." This so called 'historical-genetic-principle', see e.g. Schubring (1977) for an extensive description, came up at the turn of the 19^{th} into the 20^{th} century from biology as a result of Haeckel's biogenetic law: 'ontogenesis recapitulates phylogenesis'. This law states that in a brief period the development of the embryo of an animal recapitulates the historical development of its ancestors. Transferred to the realm of pedagogy the law implies that the genesis of knowledge in the individual follows the same course as the genesis of knowledge in the race. It appears that it is the same in the development of the mind. The development of the mathematical understanding of an individual follows the historical developments of mathematical ideas. The task of education is to make the mind of the pupil go through what his earlier generations have experienced, to pass rapidly to certain stages, but not to omit any. This is the most natural way for pupils to learn mathematics.

The desirability or even the necessity of **teaching and learning mathematics along the line of its historical development** is under discussion in several articles. According to Byers (1982) the usefulness of the 'historical-genetic-principle' is examined and it is found that it cannot be taken literally: "No one has ever suggested that a child should be kept away from the concept of zero until he has completed the study of Greek geometry in which the concept does not appear", see also Scheid (1993). Moreover Byers states that "it is hard to visualize a mechanism which would account for a correspondence between the cognitive process of attainment of mathematical concepts and their evolution as cultural product". Bkouche (1997) also discusses the analogy of the development in science and in individuals, which leads to Piaget and a critical discussion of his work.

Another statement is explained by Rogers (1997): "The error of claiming some kind of 'parallelism' between the difficulties that students might have today and the apparent difficulties of people conceptualising mathematical ideas in the past is a consequence of two things; identifying a particular idea of today as 'the same' as what we claim to be a similar notion in the past, and neglecting the process of interpretation and re-interpretation, located in a series of different socio-cultural contexts, that has occurred from that time to this". Radford (1997) and Schubring (1977) also pointed out the importance of social aspects which undermined the idea of 'parallelism'.

Berté (1995) also discusses the order of the presentation of mathematical concepts. He describes two orders: from the genesis of knowledge through history and culture or with the genesis of knowledge coming from the learner's problematical use of mathematics. She gives some reasons for choices that can be made.

On the other hand we see people who adhere to a somewhat restricted form of the 'historical-genetic-principle'. For example, Ernest (1994) says that the history of mathematics cannot absolutely dictate the necessary order in the learning of mathematical concepts. He defends the use of the history of mathematics to provide a genetic epistemological analysis of mathematical concepts for psychological and didactic purposes. According to Schubring (1988) there is a connection between students' mistakes, cognitive obstacles, and problems in the historical development of mathematics. Knowledge of the important moments of history can thus provide teachers with a tool for anticipating epistemological obstacles in the learning of mathematics, see also Van Looy (1980), Struve (1996) and Waldegg (1997). It helps the teacher to better understand errors and misconceptions in certain topics and thus can help to explain what today's pupils find hard, according to Sfard (1994).

At this point we also want to refer to the report *History in mathematics education: the ICMI study*, Fauvel/Van Maanen (2000), mentioned in the introduction. Chapter 5 from Radford gives a comprehensive analysis of the relation between how students achieve understanding in mathematics and the historical construction of mathematical thinking.

Several authors mention the necessity of teaching the history of mathematics at teacher training courses as a wish to influence teachers' attitudes and to **enrich the didactical repertoire of the teacher**. Van Maanen (1997) argues that "a look at 'old methods' can help teacher and students to evaluate their standards, to step away for a while from 'just doing mathematics' to thinking and speaking about what they are doing, and then to step back to doing, but now doing it more deliberately". Also Jahnke (1996) stimulates an attitude of reflection on one's own ideas about mathematics and mathematical methods. History of mathematics enlarges and deepens the understanding of certain topics and presents a lot of explanations, examples and alternative approaches to certain topics, see also Swetz (1995a). Barbin (1994), (1996) and (1997) argues that history of mathematics changes the epistemological concepts of mathematics by emphasising the construction of knowledge out of the activity of problem solving. Reading old sources gives a better insight in the essence of what mathematics is and improves one's didactic skills as a teacher.

2.1.2. *Relevant to pupils*

Knowledge on the side of the learner **about how mathematical concepts have developed** helps pupils' understanding, see e.g. Van Breugel (1987) Showing that mathematics is something which has been invented by people at particular stages of history, not something which has always been there, makes mathematics more concrete and gives pupils more insight. For example, through re-examining the development of known and taken-for-granted mathematical concepts, methods or proofs pupils can see that in former days mathematicians now considered as being outstanding also had their doubts and made mistakes, see Arcavi (1991) and Ofir (1991). Pupils derive comfort from realising that they are not the only ones with problems so that they get less discouraged by misunderstandings and mistakes. Besides, by comparing ancient and modern techniques pupils become aware that methods are changing and they can see that improvements in formats have made it easier to learn mathematics, see Kool (1998).

History of mathematics can also **help pupils to learn in a non-linear way**. The development of mathematical ideas proceeded not as smoothly as modern textbooks mostly suggest, see Grootendorst (1982). 'Mathematics-as-an-end-product' as presented in textbooks can be very different from 'mathematics-in-its-making'. Most mathematical ideas have never been presented in a book in the way they were discovered. When a problem had been solved the solution was turned into a theory which teachers taught without reference to the problem that laid behind. So the order was reversed, a process which Freudenthal (1993) has called 'anti-didactical inversion'. On the one hand this re-organisation seems necessary since it avoids an illogical order and the long way of the development with all its gaps. On the other hand, motivations for the development and any doubts along the way remain hidden under a deductive body of knowledge.

History of mathematics can also help to **set out a learning track in which learning obstacles and smooth progress are in balance**, see Siu/Siu (1979). It helps the learner to acquire a balance between 'rigour' and 'imagination', so that a pupil will not jump to unfounded conclusions hastily and will be given the opportunity to pursue creative thinking in different directions, which is the natural way the mind operates. Horak and Horak (1981) add that historical endeavours lead to a better understanding of the theoretical foundations of mathematics through the use and application of technical skills. Byers (1982) and Ransom (1991) argue in a similar way that historical problems provide alternative methods of solution and make pupils think.

2.2. *(Multi-) cultural arguments*

2.2.1. *Relevant to teachers*

History of mathematics can help to **develop a (multi-) cultural approach in the classroom**. Mathematics in its modern form is mostly viewed as a product of western culture. Through the study of history less known approaches to mathematics, ethnomathematics, which appeared in other cultures, can be considered, see e.g. Barta (1995) and Katz (1994). In some cases this may help teachers in their work with multi-ethnic classes, in order to re-value local cultural heritage. This approach can help to develop tolerance and respect among fellow pupils. Lehmann (1988) mentions that history gives rise to the consolidation of a scientific world view. This argument views learning in a broader sense than mathematics only. Katz (1997) provides specific examples of using the cultural context of mathematics as an instructional strategy.

The second cultural argument is that one should not isolate mathematics from the other subjects at school, see e.g. Grugnetti (1994) and Proia/Menghini (1984). Subjects are to be taught in such a way that pupils see their interconnection and mutual influence. History of mathematics provides **opportunities for cross-curricular work between mathematics and other disciplines**, for example physics and astronomy, see Bkouche (1990). A possibility to preventing the isolation of mathematics from other school subjects is by connecting it with its cultural heritage, see Nöbauer (1981), Portz (1991) and Scriba (1983). History also exposes relations among different mathematical domains.

2.2.2. *Relevant to pupils*

History of mathematics can help to **explain the role of mathematics in society**. Mathematics is a human and dynamic activity influenced by social and cultural factors. Knowledge of the sources of mathematics is therefore an integral part of the knowledge of mathematics, see e.g. Grattan-Guinness (1977), Van Looy (1980), Ofir (1991), Scriba (1983), Swetz (1984) and Windmann (1986). Rickey (1996) points out the teacher's responsibility: "As teachers of mathematics, and even more so as historians of mathematics, we are the carriers of the mathematical culture. It is our solemn responsibility to transmit this culture to our students". Pupils can be given the opportunity to see that mathematics is driven not only by utilitarian reasons, but also developed for its own sake, motivated by intellectual curiosity, recreational purposes and aesthetic criteria.

History of mathematics also **presents the development of mathematics as a human activity** and not only as a system of rigid truths, see e.g. Kronfeller (1997). This has a stimulating influence on pupils and teachers themselves. At the HIMED'90 (HIMED: History in Mathematics Education) conference one of the questions posed was if mathematics should develop a more human face. The answer was positive, see Fauvel (1991), Ofir (1991) and Russ (1991). In some cases showing the human aspects of mathematics was seen as a goal, in other cases as a consequence of teaching in historical perspective, see Heiede (1992).

Some authors mention the need to bring biographies into the mathematical classroom when humanising mathematics. They mention stories about the life and work of great mathematicians, 'heroes and heroines', see e.g. Bidwell (1993), Lightner (1991) and Ponza (1998). History of mathematics also shows that not only male but also famous female mathematicians are known. This may especially stimulate girls in learning mathematics. It is necessary to educate girls into seeing that they as women are not 'other' to mathematics, through a historical analysis of how women's participation in mathematics has been constructed, see Downes (1997).

According to Veloso (1994), using history by telling anecdotes about mathematics only is a poor teaching method. In this case, it might be interesting to give some thought to the possibly different impact of studies about success and about failure.

2.3. *Motivational arguments*

2.3.1. *Relevant to teachers*

History of mathematics can be applied in educational situations in order to bring dynamics into mathematics teaching and **create a lively classroom atmosphere**. Lessons can be made more interesting and therefore more successful, see e.g. Perkins (1991).

The history of mathematics also gives **access to useful resource material**, see Lehmann (1992) and Russ (1991). This can stimulate the teacher's enthusiasm for a topic by providing new insights into the development of ideas.

2.3.2. *Relevant to pupils*

Exploring history of mathematics helps to **increase pupils' interest for learning**, see e.g. Byers (1982) and Siu/Siu (1979). Amazing examples, subjects with a different outlook and a look into the origins of problems, concepts, methods and proofs may intrigue and motivate pupils. They may make mathematics lessons less frightening, more enjoyable and exciting. It also enables brighter learners to look further.

Secondly, knowledge of the history of a mathematical subject **helps in understanding the subject matter itself**. Freudenthal (1981) doubts this. A historical approach to a subject only following the historical line would be much harder, especially for young children who do not yet have a historical sense and who cannot understand unfamiliar original ideas and techniques, see Grattan-Guinness (1973). A pupil confronted with original problems and techniques will sometimes have to spend hours trying to reconstruct the situation from unfamiliar ideas. Grattan-Guinness thinks that the way in which history can be applied to mathematics lessons is the way in which pupils relive creative work and imitate the individual discovery of previous results.

2.4. *Practical objections raised by teachers*

Several difficulties have been raised in integrating the history of mathematics in the classroom, see e.g. Fauvel (1991) and Fowler (1991). The first objection is that most teachers do **not have enough historical expertise**. This is a consequence of the lack of appropriate teacher education programmes for integrating the history of mathematics in mathematics lessons.

Secondly, teachers do **not have access to the right materials** to make a historical approach in their lessons possible for them. Besides, there is a lack of supporting teaching material such as didactical guidelines and empirical descriptions for teachers on how to use available historical material in their lessons.

The third objection for teachers is related to the former two: '**lack of time**'. It takes a lot of time to make historical material to use in the classroom. On the other hand, this objection is relative. Material once produced can be used year after year and it will still have its positive influence on the quality of teaching and learning.

3. *How* TO USE HISTORY

3.1. *General integration of history of mathematics in the mathematics classroom*

Answering the question *why* we should apply the history of mathematics to mathematics education and deciding *that* we should do so is one thing. Knowing *how* to do so is a different thing altogether. Inspired thoughts do not help teachers with the practical problem of how to create educational programmes. One has to make choices about the mathematical educational purposes, the illustrative period of history, the non-mathematical goals to be achieved, the didactical tools and so on. Furinghetti (1997) gives some examples and considers how the domains of history of mathematics may interact in the process of mathematics teaching. She focuses on the teachers' role and explores the relationship between research in mathematics education and the history of mathematics.

For integrating the history of mathematics in classroom **giving historical information to inform about a historical period or mathematical topic** can be used corresponding to motivational and (multi-) cultural arguments. For example, secondary sources, such as textbooks with historical narratives, can provide an introduction to concepts which are new to pupils. Deeper awareness of the social and cultural contexts in which mathematics has been done can be developed. Secondary source material also tells us stories about past mathematicians. A biography for example gives the name of famous mathematicians, dates of important events and tells about famous works.

According to conceptual, and also motivational arguments, **a teaching and learning approach inspired by history to introduce mathematical concepts or methods** can be followed. An example is given by Furinghetti and Somaglia (1998):

– to work at an informal level using colloquial language, graphical presentations, diagrams to stimulate pupils' 'intuitive ideas' on a certain concept
– to exploit pupils' 'intuitive ideas' stimulated in the preceding stage to outline the main features of the concept
– to introduce a mathematical formalisation of the concept.

The history of mathematics has revealed itself to be very suitable for this purpose. It provides a lot of contexts in which pupils' 'intuitive ideas' may arise. These contexts can be found in primary and secondary sources.

Another example is given in Arcavi (1987). It gives a general framework for an activity that can be developed around a primary source:

– 'dictionary questions' that help one to become acquainted with unknown notations, symbols, names of concepts, or formulations in the source
– redoing the mathematics in modern notation, leading to an understanding of what was done
– applying the operation or process to other examples
– discussing the mathematics involved with our hindsight (justifications, generalisations, etc.).

Primary source material, such as excerpts from original mathematical documents, provide a huge amount of relevant and interesting problems. It provides opportunities for investigations and opportunities to speak with pupils about creativity in mathematics and alternative methods of solutions to problems.

But the analysis of historical texts and making the texts accessible to students is a difficult activity. The crucial steps of the historical documents have to be identified and reconstructed so that they become appropriate for classroom use. An important aspect is to find suitable questions so that students become deeply involved in the historical context under study.

To understand the intention of the original author demands a feeling for the intellectual, social and cultural context in which the source has been written. The modern reader should be aware of the hypothetical and intuitive character of his or her interpretation. There is a risk that the history of mathematics is misrepresented. Heiede (1996) gives a long row of examples of such misrepresentations.

According to Heiede, we must really guard against uncritically accepting misrepresentations. But we also must realize that we cannot guard ourselves completely. It is very difficult to keep complete historical correctness in bringing history of mathematics in the classroom, because teachers of mathematics are not all professional historians of mathematics.

For an extensive description of the use of original sources in the mathematics classroom we refer to Radford's chapter 9 of the *The ICMI study*, Fauvel/Van Maanen (2000).

Next to primary sources, original instruments can be introduced in the mathematics classroom. It is possible to illustrate mathematical concepts and proofs using instruments that have been devised for this purpose. Examples of such instruments are tools for drawing conic sections and the trisectrix- and quadratrixcompasses for solving ancient Greek geometrical problems.

Finally we want to refer again to the *The ICMI study*, Fauvel/Van Maanen (2000). Chapter 7 from Tzanakis and Arcavi gives an analytic survey of how history of mathematics has been and can be integrated into the mathematics classroom. Next to the examples mentioned above the following possibilities are set out: research projects based on history texts, worksheets, historical problems, experiential mathematical activities, plays, films and other visual means, outdoors experiences and the World Wide Web.

3.2. *Experiments in classrooms*

Some authors published their experiments in which they translated their theoretical ideas and historical knowledge into practical classroom lessons. Ransom (1993) and (1995) for example gives a description of the use of old surveying and navigational instruments in the classroom. He gives an extensive instruction, from historical and mathematical point of view, how to use easily made instruments such as the cross staff, the astrolabe and sundials. Teaching geometry in this way involves pupils in the practical uses of geometry and provides a motivating force for both practical and theoretical geometry.

Bartolini Bussi et al. (1999) analyse a teaching experiment in primary school in the field of experience of gears, which are part of everyday experience from very early childhood. The recourse to this 'real' context motivates pupils to learn geometry. The aim of the project is to identify the characteristics which have enabled the pupils to approach theoretical thinking, and in particular mathematical theorems. By means of Euclid's geometry and Heron's kinematics mathematical modelling of activities with gears is accomplished. The paper presents the findings of the teaching experiment at the level of interpersonal classroom processes and of individual mental processes. Douek (1999) also describes a classroom experiment in primary school in which modelling in a 'real' context takes a central place. She tries to explain a schoolfellow's mistake in the use of a geometrical scheme of the sunshadows phenomenon.

Another concrete idea was put forward by Brodkey (1996), who founded a Euclid-club. Some twenty persons, teachers and students of a High School, volunteered to study book I of Euclid's *Elements* together. They interacted as equals and were alternately standing before the group to demonstrate a proposition. Afterwards they discussed the ideas. They did it purely for the pleasure of learning. Brodkey describes that students, by working through each proof, deepened their understanding of geometry. They also improved their speaking abilities and discussion skills and were exposed to sophisticated logical arguments. Studying and intensively discussing a primary source brought pleasure to the participants.

Laubenbacher and Pengelley (1996) write about this same method in which students read original texts without any modern writer or instructor as intermediary or interpreter. They observed a special excitement which resulted from reading a first-hand account of a new discovery. Van Maanen (1997) and Thomaides (1991) describe similar processes. Although they notice that the pleasure of studying primary sources is motivating and stimulating to the students, they emphasise the positive effect on the learning of mathematics. This improvement is caused by more fun, by livelier lessons, by using the comparison of old methods with modern techniques in order to value the power of our present-day methods and by a form of didactics that requires more creativity on the part of the pupils.

In some articles the key-word in creating resource material seems to be 'reinvention'. This word was coined by Freudenthal (1973) who considered learning mathematics as a process in which the student invents what others have invented before him or her. While studying resource material pupils have to go through 'old' steps, but the teacher, for example, could translate some important steps in modern form in such a way that pupils could make their own inventions.

Artmann and Seeger (1982) write about three lessons they gave on irrationals and geometry in this manner. They handed out worksheets to the pupils and expected them to work through these themselves and to go through 'old' steps. The lay-out of the sheets was modern without using primary sources. Afterwards the pupils and the teacher together discussed the striking discoveries. Gerstberger (1986) puts forward ideas in this manner. He looks for the relevant steps to be taken by his pupils in such a way that they can make their own 'reinvention' about the Pythagorean theorem. He hopes the pupils will have a deeper understanding of the subject and they will notice that important mathematical theorems are sometimes hard to find.

Makowski and Strong (1996) did an experiment with geography students on earth measurement, based on the work of Eratosthenes. They explain how teachers can use Eratosthenes' method in the classroom to calculate the size of the earth. With the help of a guide, students made their own calculations and in a debriefing the results were checked and worked out. According to the authors this experiment shows students the principles of abstraction from the physical into a mathematical relationship.

A somewhat different opinion we encounter in Windmann's (1986) idea. He suggests that teachers select historical texts in which students can, after careful reading, ask their own questions. Therefore they have to read an original text in their own language. If necessary, the teacher has to translate the text.

4. Specific geometrical subjects

This section reviews many more, more or less, concrete articles in order to show the diverse ways in which the history of geometry is used in teaching and learning geometry. The presentation follows a subdivision into geometrical subjects, along the ideas of the Dutch mathematics-innovation team W12-16. We added one subject category ourselves: 'geometry and physics' because of the strong, remarkable and also historical relationship between geometry and physics, which often seems to be ignored while teaching these two disciplines.

The division of the presentation is as follows:

- Computations in geometry: length-, surface-, volume-computations, proportions, trigonometry.
- Geometrical constructions: constructions with ruler and compasses or other instruments.
- Properties of two- and three-dimensional figures: similarity, congruence, proportions.
- Vision geometry[2]: perspective, two-dimensional sections of three dimensional figures, nets.
- History of geometry: biographies, dialogues, stories, anecdotes.
- Geometrical loci: co-ordinate systems, vectors.
- To argue and to prove: proofs, structures of geometrical proofs, knowledge of the concepts: definition, theorem, axiom.
- Geometry and physics.

Representative articles in each category are described. Other articles are only mentioned in appendix A. To avoid a large amount of detailed information we shall briefly describe the main subject of the articles. Table I to VIII from appendix A will give additional, structured and easy-to-access information for the use of practising teachers and educational researchers. This provides the possibility to approach the literature with a thematic starting point. In our view disclosing historical information for teachers is crucial.

Appendix B gives a survey of articles that consist of historical essays. These can be very useful to teachers who want to be informed about a historical period or mathematical topic before teaching the topic.

4.1. *Computations on geometrical subjects*

Much has been written about the Pythagorean theorem. For example, Swetz (1977) shows the technique of piling up squares, which makes use of an intuitive approach to solve algebraic problems. This technique was used in ancient Greece and Babylon but, according to Swetz, was developed into a high art in China. Burns (1997) describes the teamwork he organised around a Babylonian clay tablet. His only question was: "Here was a piece of what appears to be mathematics, from the Babylonians of ca. 1750 BC. What could it be about?". His experience was positive. Most of the groups worked seriously on this challenge and presented clever answers of their own to the question. But some remarkable differences revealed. Some pupils discovered what is was about without realizing what they had achieved, other pupils gave the 'right' answer by using a book to learn what all the numbers were, without gaining any idea what the tablet was about. MacKinnon (1992) also uses this and other Babylonian clay tablets of the Pythagorean theorem.

Finding the volume of a sphere requires knowledge of the formula $V = 4/3\ \pi\ r^3$. The history of this subject can be used to justify this formula. One way to do so is using Kepler's method, which offers an insight into definite integration. In this way, teaching the volumes of objects can also be applied as an introduction to calculus, see Tobias (1981). The method by means of which Archimedes found the volumes of various geometrical objects can be applied to prepare the integration process as repeated summation. Archimedes balanced the weights of a certain sphere, cylinder and cone. Eagle (1998) describes the steps in which she led her students through this work. The work of Archimedes also inspired scholars to have a look at the surface area formula for the sphere through working with projections of the globe on a map, see Mackinnon (1989).

The circumference of a great circle of a sphere can be taught through earth measurement. Führer (1991) uses the story of Eratosthenes for his pupils to sketch a framework for the meaning and value of the computation of the circumference of the sphere. He suggests the development of resources to let the students work on the subject in 'guided reinvention'.

Computing the surface of a rectangle with the help of formulas that involve variables is quite normal for mathematicians who live in the Cartesian era of mathematics. But in secondary school pupils often have trouble in working with unknowns and prefer a more intuitive geometrical approach. Van Maanen (1998) reports that school children recognise their own intuitive thinking in the 17th century work of Cardinael, as far as problems of battle-arrays are concerned. He lists some authentic 17th century problems for use at school.

Another geometrical computational topic is the approximation of π. Ofir (1991) suggests that, for motivational reasons, sources may be used from the home country of the pupils. He mentions for example the work of the Hebrew Maimonides (12th century) for an Israeli school. Corris (1990) demonstrates how pupils can make their own approximations of π with the help of construction and computation exercises inspired by the work of Archimedes. Führer (1991) mentions Archimedes' work in combination with the work of Viète and Descartes in order to introduce a notion of approximation. He argues that exploring their works can inspire teachers and let them return refreshed to the classroom.

4.2. *Geometrical constructions*

The ancient Greek tradition of constructing geometrical figures by ruler and compasses only offers a good opportunity to let pupils work through old methods. They can follow the descriptions from old sources and rediscover geometrical facts in an intuitive way. The construction of regular polygons is mentioned in several articles. Hogendijk (1996) describes a construction of a pentagon from an old Persian manuscript. Pupils can check this construction method themselves by following the description of the (translated) manuscript. Persian manuscripts often mention the construction of various patterns and figures as well in order to create mosaics. Hogendijk thinks that these mosaics give an opportunity to let pupils work with geometrical constructions.

Next to ruler and compasses, some authors discuss other drawing instruments such as the parallel ruler and the conic section drawer. In the 17th century the interest in instruments for drawing conic sections increased, according to Van Maanen (1992b). Pupils can study these construction instruments by looking at old figures from 17th century sources. Van Maanen also gives an example of working on constructions by means of ruler and compasses as in old times. Although the purely historical article on the parallel-ruler by Impens (1988) does not pay attention to educational situations, teachers can still make good use of it because of his extensive description of types of constructions.

Another typical instrument is the trisectrix- or quadratrix-compasses. With this instrument two of the three 'classical' problems, the trisection of the angle and the quadrature of the circle, can be solved. Hischer (1994) describes how merely the idea of this instrument and its result, the trisectrix-curve, suffices to solve the problems. His suggestion is to teach this subject in the upper classes of high school.

4.3. *Properties of geometrical figures*

Lumpkin (1978) presents the solution of cubic equations by the Khayyam method, 11^{th} century, which uses the well-known properties of intersecting conic sections. She argues that students will gain greater insight into the geometric nature of conic sections through their work on this subject, because Khayyam uses geometrical methods for the solution of cubic equations.

4.4. *Vision geometry*

Articles that belong to this section deal with the interpretation of geometrical objects, such as perspective problems or the use of geometrical figures in architecture or models for building polyhedra. An experiment on the question: "Why do we find the elliptical form in architecture only in the baroque period?" was asked in a senior high school class and is described by Proia and Menghini (1984). A major outcome was the conviction in both teacher and students not to dismiss the possible contributions given by other branches of science to the development of mathematics.

Bartolini Bussi and Mariotti (1999) discuss the shape of particular sections of a right cone and a right cylinder and whether either is egg-shaped. Students try to find historical arguments to help in harmonising the figural and conceptual aspects of the problem.

4.5. *General education*

Showing students a concrete example of progress and continuity in mathematical history is proposed by Lumpkin (1978). She uses modern analytic geometry to analyse the Arabic-Greek method of solving cubic equations by intersecting conic sections, see subsection 4.3. The purpose in this case is a mathematical one: to gain greater mathematical insight. Showing pupils the dynamics of the mathematical development helps them to reconsider their belief of a static mathematical branch and will get pupils to appreciate the fact that mathematics is a product of the human mind. Lightner (1975) discusses in this connection biographical facts of early 'giants' of geometry: Thales, Pythagoras, Plato, Euclid and Apollonius.

Another advantage of using the history of a subject is the large amount of ‘old’ problems. Lehmann (1992) offers 25 ‘old’ mathematical problems, which he gathered from various sources, for the pleasure of thinking. Baptist and Diener (1988) take the view that a historical problem offers students a good opportunity to get the experience of changing their perspective in solving problems, because of the variety of methods from various periods that have been applied to solve them. They present as an example the two-tower-problem and the four different solutions of Leonardo of Pisa.

A purely historical article describing the level of knowledge on spherical trigonometry in the days Columbus we find in Edwards (1992). A different type of article related to the history of geometry we find in the work of Strecker and Rickey. They both discuss the measurement of the size of the earth. Strecker (1998) comments on a historical mathematical exercise on measurement of the sphere from a schoolbook that is based on the work of Eratosthenes. Because of the importance of a critical attitude of pupils to the things they learn he suggests a lot of critical questions one can ask about such an exercise. His questions stimulate a deep consideration of this kind of historical exercise. Rickey (1992) describes the mathematical knowledge Columbus based his voyage on. He mentions to his students the work of Eratosthenes and he also suggests that it is worthwhile discussing the effects of errors of measurements. In his article he makes proposals to investigate this subject.

4.6. *Geometrical loci*

Van Maanen (1992a) developed a project on the division of alluvial deposit. Eleven years old pupils studying Latin and mathematics studied a Latin treatise by Bartolus of Saxoferrato dated from 1355. Bartolus proposed the mathematical criterion that new land will be the property of the owner of the nearest old land, which leads into the mathematical theory of perpendicular bisectors. Besides integrating two subject in the same project, it was a way of encouraging pupils to work together, to see the importance of mathematics in society and to discover ruler-and-compass constructions.

4.7. *Arguing and proving*

Arsac (1987) studies the historical origins of mathematical proof, beginning in Greece in the 5th century BC. In his paper he discusses the question: Is the appearance of mathematical proof linked to a particular problem within mathematics or is it a consequence of the general course of Greek thought?. Horak and Horak (1981) show in what way the Greeks manipulated geometrical shapes to carry out algebraic operations and to prove algebraic identities. In their opinion this might be used in classrooms to provide mathematical understanding and enrichment. Artmann (1991) also emphasises this statement and adds that Greek and Babylonian methods offer the opportunity to show how a given problem can be solved by different, but isomorphic, methods. He suggests three kinds of lessons and gives a reasonable amount of background information. In a purely historical article Deakin (1990) describes the history of proofs of the theorem that in an isosceles triangle the base angles are equal. He starts with Euclid's proof, commonly known as 'pons asinorum', and ends with modern proofs.

4.8. *Geometry and physics*

Tzanakis (1999) gives two examples of a genetic approach revealing interrelations between mathematics and physics: firstly Newton's gravitational law derived from Kepler's law and secondly the foundations of special relativity theory as an example of the use of matrix algebra. He describes both theory and exercises to use in high school and with 1st-year undergraduate students. In an article of Tzanakis and Thomaides (1998) the connection between mathematics and physics is discussed and followed by a few concrete examples.

4.9. *Recurrent points of interest*

Teaching geometry with the history of mathematics as a didactic tool requires specific choices. Choices about the *aims* the teacher intends to achieve and choices about the *didactics*: In what way do we teach? What method do we use? And how can we make or find resources that fit in and support this choice?

In most articles the educational aim is an improvement of pupils' understanding of mathematics. Many other authors suggest the use of the history of mathematics for its own sake. For example in the articles on constructions it seems to be the construction itself that takes a central place. To go through an old source and rediscover geometrical construction facts appears to be done because of the beautiful geometry.

In half of the articles the authors do not explain what their resources look like. The way in which the history appears in resources varies from extracted primary sources, via descriptions of 'old' mathematics in modern language, to anecdotes or tales. But in most of the cases an old text or illustration occupies the central position. Rarely is a general survey of the history given to pupils. It is more common to focus on a specific problem.

Only a few authors describe in some detail what their didactic approach was. The way teachers let their pupils study a source differs, but mostly it is a self-inventing strategy. Sometimes self-study means completely free reading of a text, individually or in a group, asking one's own questions, sometimes it consists of reading a text, guided by teacher-questions, sometimes in explaining an original source to others after a thorough preparation and sometimes it consists in literally following the steps in a source and repeating them. What these descriptions have in common is an appeal to the pupils' self-motivation: not a passive, but an active way of learning, struggling with a text or an old problem.

Several authors mention the advantage of an intuitive geometrical approach in 'guided reinvention'. In our opinion we have to ask ourselves if we can speak of a true 'reinvention' when pupils study old texts or old problems. Obviously, the invention has occurred before the mathematician writes his text. This text only consists of the results; the process of invention itself is rarely described. For example, when pupils study an old Egyptian text in which the area of a circle is calculated via $(8/9\ d)^2$, with d diameter of the circle, do they really invent the way of area-measurement or do they learn about an old way of doing so? We do not know how the Egyptians discovered their result, but if we assume that they tried to approximate the circle with several squares, do we not give our pupils a better chance to invent something, if we let them struggle with squares and circles instead of studying old texts? Or should we combine these two activities? We have not found an explanation or a legitimation of the chosen didactic method anywhere. Therefore this question cannot be answered on the basis of the articles surveyed. It could, and should, be part of further investigations.

5. Conclusions

Especially in the nineties the interest in the use of the history of geometry in mathematics education has increased. We found a large amount of different arguments that plead for the use of the history of mathematics as a didactic tool, but the contributions to the discussion seem to be isolated from each other. The amount of general articles that contribute to the debate outnumbers the practical essays which contain suggestions for resources or lessons.

Teaching geometry in historical perspective seems to offer a possibility to develop resources that are in accordance with the natural development of the pupils concerned. For example, computations on geometrical objects nowadays have become algebraic activities. These do not follow pupils' intuitive experience with and thinking about these objects. Old Greek manipulations with geometrical shapes to prove algebraic identities can be used to make exercises more recognisable to pupils to provide mathematical understanding.

Another conclusion is that a gap exists between historians, writing 'general' articles, and teachers, writing 'practical' articles. Most of the essays lack a legitimation of the ideas and suggestions. For example, the following questions have hardly been answered: What makes one think that the use of history deepens the mathematical understanding? Is it really motivating to stress the human aspect of mathematics or is it the enthusiastic teacher who motivates his class? Has any research been done to confirm these previous thoughts? Is there any psychological theory to confirm it? And how do people justify their choice of resources? If we could answer these questions it would probably be easier to generalise the various different ideas. With such a generalisation we could probably develop a sort of 'checklist' to be consulted if we decide to teach with the history of mathematics as a didactic tool.

Especially the lack of research into the methodology of teaching mathematics with the history of mathematics as a didactic tool has to be noticed. The didactical strategies used are based on personal preferences and in most cases the authors do not use didactic or methodological arguments in support of their ideas. In order to develop the implementation of the history of mathematics in mathematics education we need to know more about the methodology. Questions such as: How do we give pupils a real chance to reinvent something? In order to do the 'reinvention' of a specific geometrical subject which sources are useful and which are useless? Are the results of organising the 'reinvention' of one specific geometrical subject generalizable and can they be applied to other domains of geometry? And can we draw up a list of general requirements for doing the 'reinvention'? have to be answered.

FUTURE RESEARCH

It is our intention to continue our research in this area. Our aim is to translate the results of this study into resource material that will be used in geometry lessons in school and to write an instruction for the teacher on 'how to use this material'.

We are also going to design a method for evaluating the effectiveness of mathematics teaching which uses classroom material in which history is an integral part. The produced classroom material will be tested according to the designed method and conclusions will be drawn. The conclusions may help teachers, teacher trainers, textbook authors and those who work on curriculum development to determine in what ways and to what extent history may be used in teaching mathematics.

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NOTES

1. Journal for Research in Mathematical Education, (American), '93–'99; Mathematics Teacher, (American), '95–'99; Wiskunde en Onderwijs, (Belgian), '76–'99; For the Learning of Mathematics, (Canadian), '85–'99; Euclides, (Dutch), '70–'99; De Nieuwe Wiskrant, (Dutch), '75–'99; Educational Studies in Mathematics, (English and French), '76–'99; International Journal of Mathematical Education in Science and Technology, (English), '80–'99; The Mathematical Gazette, (English), '79–'99; Mathematics in School, (English), '87–'99; Science and Education, (English), '92–'99; Recherches en Didactique des Mathématiques, (French), '94–'99; Repères IREM, (French), '94–'99; Mathematik Lehren, (German), '83–'99.
2. Vision geometry, according to De Moor (1999, p. 691), is based on looking at, perceiving, representing and explaining spatial objects and spatial phenomena, in which the idea of the straight line as a vision line (sighting) and a ray of light plays a central role.

APPENDIX A

In the tables one can find articles concerning one subject-category, corresponding to the subsections of section 4. In each table, the different columns provide information on the subject, the historical period or mathematician under consideration, the amount of background information, the didactics, the resources, the practical usefulness for teachers and the reference to the author(s) of the article. The aspects of background information, didactical method, resources and usefulness are placed on a scale which will be explained:

– Background information, scales 1,2,3
(Remark: comparisons between the articles with this scale-division have to be carefully made, because we placed the articles onto this scale in relation to the aim of the article.)

1:= the author presents little information about the history involved. If the readers do not have knowledge of the subject, they have to acquaint themselves with the original text and/or secondary sources, before teaching.
2:= the author presents information about the history involved and this information will probably be enough to use the suggestions in one's own mathematics lessons; the article does not provide a survey of the historical period or subject.
3:= there is a reasonable amount of information in the article so that one can get acquainted with the subject and feel comfortable while teaching it.

– *Didactical method, scales 1, 2, 3*
1:= the teacher lectures to his class; no practical exercises are mentioned.
2:= classical work; teacher and pupils discuss a subject together.
3:= group work, with the help of an (extended or short) written guide.

– *Resources, scales 1, 2, 3, 4, 5*
1:= written guide with modern exercises.
2:= written guide with 'old' problems translated into modern mathematical language.
3:= written guide with parts of an original text.
4:= translated original text, e.g. Greek into modern English.
5:= original text.

– *Practical usefulness, scales 1, 2, 3, 4*
1:= historical article.
2:= the article contains no suggestions for resources, except the remark that the subject discussed could be used in teaching geometry. Teachers still have to spend a lot of work and time on preparing their lesson.
3:= the article contains suggestions of how to teach; these still have to be worked out to create resources.
4:= the article contains ready-made resource material, e.g. worksheets.

TABLE I

Corresponding to section 4.1: computations on geometrical subjects

Subject	Applied history	B I	D M	R	U	Reference
Volume of a pyramid	Moscow Papyrus, ca. 1650 BC	1	–	–	2	Barnes/Michalowicz (1995)
Volume of a sphere	Archimedes, ca. 250 BC	2	–	–	2	Eagle (1998)
Volume of a barrel	16th C. examples	2	–	–	2	Meskens (1992)
Volume of a sphere	Kepler, 17th C.	1	3	1	4	Parker/Straker (1989)
Volume of a sphere	Chinese mathematics, ca. 300 BC-500	2	–	–	2	Swetz (1995b)
Volume, various objects	Archimedes, ca. 250 BC	1	1	–	3	Tobias (1981)
Surface area under a hyperbola	Gregory of Saint-Vincent, 17th C.	3	–	3	2	Dhombres (1993)
Surface area of a triangle	Frans van Schooten, 17th C.	2	3	4, 5	4	Van Maanen (1997)
Surface area of a square and a rectangle	Cardinael, 1610	2	3	4, 5	4	Van Maanen (1998)
Surface area of a sphere	Archimedes, ca. 250 BC	1	–	–	2	Mackinnon (1989)
Circumference of a sphere	Eratosthenes, ca. 300 BC	2	–	–	2	Führer (1991)
Circumference of a sphere	Eratosthenes, ca. 300 BC	3	3	2	3	Makowski/Strong (1996)
Pythagorean theorem	Several sources	3	3	3	4	Anonymous (1996)
Pythagorean theorem	Babylonian mathematics, ca. 1750 BC	1	3	5	4	Burns (1997)
Pythagorean theorem	Several examples, period BC	1	2, 3	–	3	Gerstberger (1986)
Pythagorean theorem	Several sources	1	1, 3	1	3	Kindt (1979)
Pythagorean theorem	Babylonian mathematics	1	2, 3	2, 3	3	MacKinnon (1992)
Pythagorean theorem	Chiu-chang suan-shu, ca. 300 BC	2	3	2, 3	4	Swetz (1977)
Trigonometry	Ptolemeus, ca. 150	1	–	–	2	Bidwell (1993)
Calculating π	Archimedes, ca. 250 BC	1	3	1	3	Corris (1990)
Calculating π	Legendre, ca. 1800	2	2, 3	3	3, 4	Métin (1997)
Calculating π	Several sources	1	–	–	2	Ofir (1991)
Several computations	Kepler, ca. 1600	1	2, 3	2	3, 4	Bero (1993)
Several computations	Several sources	2	2, 3	2	4	Johan (1996)
Several computations	Babylonian and Chinese mathematics	2	2, 3	3	3	Swetz (1989)
Several area and volume computations	Pappus, ca. 320	1	–	–	2	Turvey (1995)

Note: The aspects background information (BI), didactical method (DM), resources (R) and practical usefulness (U) are placed on a scale on the following way: B I – 1: little information, B I – 2: enough information, B I – 3: a lot of information. D M – 1: teacher lectures, D M – 2: classical work, D M – 3: group work. R – 1: guide with modern exercises, R – 2: guide with old problem in modern mathematical language, R – 3: guide with parts of an original text, R – 4: translated original text, R – 5: original text. U – 1: historical, U – 2: no resources suggestions, U – 3: suggestions of how to teach, U – 4: ready-made resources.

TABLE II

Corresponding to section 4.2: geometrical constructions

Subject	Applied history	B I	D M	R	U	Reference
Constructions of patterns and mosaics	Several sources	1	3	–	3	Ernest (1998)
Construction of a regular pentagon and mosaics	Middle Ages	2	3	2	4	Hogendijk (1996)
Constructions of a square with a given area	10^{th} C.	1	–	2	4	Laforce (1989)
Section of an angle	Descartes, 17^{th} C.	2	3	3	4	Le Goff (1994)
Constructions of a bisector	Marolois, 17^{th} C.	2	2, 3	3	4	Van Maanen (1997)
Squaring of curvilinear figures	Hippocrates of Chios, 5^{th} C. BC	2	2, 3	2	3	Stoll (1998)
Trisection of an angle	Archimedes, ca. 250 BC	2	2, 3	2	3	Thomaides (1991)
Trisection of an angle	Snell	1	2	2	2	Toumasis (1995)
Constructions with the trisectrix and quadratrix	Hippias, ca. 460 BC; Dinostratos, ca. 350 BC	2	–	–	3	Hischer (1994)
Constructions with the parallel-ruler	Several sources	3	–	–	1	Impens (1988)
Instruments for drawing conic sections	Frans van Schooten, 17^{th} C.	2	3	2, 3	4	Van Maanen (1992b)

TABLE III

Corresponding to section 4.3: properties of geometrical figures

Subject	Applied history	B I	D M	R	U	Reference
Geometrical proofs of algebraic identities	Omar Khayyam, 11^{th} C.	2	–	–	2	Lumpkin (1978)
Similar triangles	Marolois, 1629	1	–	4	2	Van Maanen (1998)
Orbits of circles and ellipses	Galilei, 17^{th} C.	2	–	4	1, 3	Portz (1991)

Note: The aspects background information (BI), didactical method (DM), resources (R) and practical usefulness (U) are placed on a scale on the following way: B I – 1: little information, B I – 2: enough information, B I – 3: a lot of information. D M – 1: teacher lectures, D M – 2: classical work, D M – 3: group work. R – 1: guide with modern exercises, R – 2: guide with old problem in modern mathematical language, R – 3: guide with parts of an original text, R – 4: translated original text, R – 5: original text. U – 1: historical, U – 2: no resources suggestions, U – 3: suggestions of how to teach, U – 4: ready-made resources.

TABLE IV

Corresponding to section 4.4: vision geometry

Subject	Applied history	B I	D M	R	U	Reference
Conic sections	Witelo, ca. 1200; Dürer, 1525	2	2, 3	3	3	Bartolini Bussi/Mariotti (1999)
Conic sections	Apollonius, ca. 200 BC; Copernicus, ca. 150; Kepler, ca. 1600	2	2, 3	–	3	Proia/Menghini (1984)

TABLE V

Corresponding to section 4.5: general education

Subject	Applied history	B I	D M	R	U	Reference
A timeless problem with different time-dependent solutions	Leonardo of Pisa, 13th C.	2	–	–	2	Baptist/Diener (1988)
Spherical trigonometry	Columbus, 15th C.	2	–	–	1	Edwards (1992)
Historical problems	Babylonian mathematics until today	3	3	2	4	Lehmann (1992)
Great geometers	ca. 600–200 BC	1	–	–	2	Lightner (1991)
Geometrical proofs of algebraic identities	Omar Khayyam, 11th C.	2	–	–	2	Lumpkin (1978)
15th-century geometry	Eratosthenes – Columbus	1	–	–	1, 2	Rickey (1992)
15th-century geometry	Eratosthenes – Columbus	1	–	–	2	Strecker (1998)

Note: The aspects background information (BI), didactical method (DM), resources (R) and practical usefulness (U) are placed on a scale on the following way: B I – 1: little information, B I – 2: enough information, B I – 3: a lot of information. D M – 1: teacher lectures, D M – 2: classical work, D M – 3: group work. R – 1: guide with modern exercises, R – 2: guide with old problem in modern mathematical language, R – 3: guide with parts of an original text, R – 4: translated original text, R – 5: original text. U – 1: historical, U – 2: no resources suggestions, U – 3: suggestions of how to teach, U – 4: ready-made resources.

TABLE VI

Corresponding to section 4.6: geometrical loci

Subject	Applied history	B I	D M	R	U	Reference
Elementary constructions	Bartolus of Saxoferrato, ca. 1350	2	2, 3	2, 5	3, 4	Van Maanen (1992a)

TABLE VII

Corresponding to section 4.7: arguing and proving

Subject	Applied history	B I	D M	R	U	Reference
Origins of mathematical proofs	Greek mathematics	1	–	–	1	Arsac (1987)
Proofs of sum of the angles of a triangle	Euclid's *Elements*, ca. 300 BC; Clairaut, 1765	1	–	2	2	Barbin (1991)
Geometrical proofs	Euclid's *Elements*, ca. 300 BC	2	3	–	4	Brodkey (1996)
Geometrical proof	Euclid's *Elements*, ca. 300 BC	1	2, 3	2, 4	3	Danckwerts/Vogel (1997)
Proofs of similar base-angles of an isosceles triangle	From Pappus to today	1	–	–	1	Deakin (1990)
Geometrical proofs	Liu Hui, 3th C.	3	–	2	2	Siu (1993)
Geometrical proofs of algebraic identities	Euclid, ca. 300 BC	3	1, 3	2, 4	3	Artmann (1991)
Geometrical proofs of algebraic identities	Greek geometry, ca. 600–300 BC	2	–	–	2	Horak/Horak (1981)
Geometrical proofs of algebraic identities	*Chiu-chang suan-shu*, ca. 300 BC	2	3	2, 3	4	Swetz (1977)

Note: The aspects background information (BI), didactical method (DM), resources (R) and practical usefulness (U) are placed on a scale on the following way: B I – 1: little information, B I – 2: enough information, B I – 3: a lot of information. D M – 1: teacher lectures, D M – 2: classical work, D M – 3: group work. R – 1: guide with modern exercises, R – 2: guide with old problem in modern mathematical language, R – 3: guide with parts of an original text, R – 4: translated original text, R – 5: original text. U – 1: historical, U – 2: no resources suggestions, U – 3: suggestions of how to teach, U – 4: ready-made resources.

TABLE VIII

Corresponding to section 4.8: geometry and physics

Subject	Applied history	B I	D M	R	U	Reference
Optic	Heron of Alexandria, 1^{th} C.	2	2, 3	2	3, 4	Jahnke (1998a)
Astronomy and trigonometry	Aristarch of Samos, ca. 300 BC	2	2, 3	2	3	Jahnke (1998b)
Use of physics in mathematics	Several examples	2	–	–	1	Tzanakis/Thomaides (1998)
Gravitational law and special relativity theory	Newton, 17^{th} C.; Einstein, 1905; Poincaré, 1904	2	2	2	4	Tzanakis (1999)

Note: The aspects background information (BI), didactical method (DM), resources (R) and practical usefulness (U) are placed on a scale on the following way: B I – 1: little information, B I – 2: enough information, B I – 3: a lot of information. D M – 1: teacher lectures, D M – 2: classical work, D M – 3: group work. R – 1: guide with modern exercises, R – 2: guide with old problem in modern mathematical language, R – 3: guide with parts of an original text, R – 4: translated original text, R – 5: original text. U – 1: historical, U – 2: no resources suggestions, U – 3: suggestions of how to teach, U – 4: ready-made resources.

APPENDIX B

This appendix contains a description of historical essays. In the first table one can find articles on a historical subject. The second table consist of more or less philosophical articles with subjects related to the history of mathematics and mathematics education. We shall not describe them in detail because of the great variety of these articles. These articles, from both tables, serve as information to deepen the readers knowledge of a subject or to find ones way as a teacher in the literature on the subject. To that end, in this appendix we refer to the amount of useful bibliographic references one can find at the end of each article.

The first column of each table contains the main subject of the article; the second column refers to the amount of bibliographic references, to primary and secondary sources, and in the third column one finds the reference to the author(s).

TABLE II

Philosophical articles

Subject	Amount of bibliographic references	Reference
Mathematics and society	> 10	Bos (1990)
Socrates and Plato's *Meno*	> 10	Fernandez (1994)
Founders of mathematics	> 10	Fletcher (1982)
Historical correctness	< 5	Freudenthal (1987)
Resources for teachers	> 10	Rogers (1991)

TABLE I

Historical articles

Subject	Amount of bibliographic references	Reference
Short history of geometry	> 10	Adele (1989)
The work of Alexis Claude Clairaut	5 – 10 (*)	Beckers (1998a)
Recreational mathematics of Guyot	5 – 10 (*)	Beckers (1998b)
Dutch 17th C. mathematicians	> 10	Bissell (1987)
The three-point problem and solutions in history	5 – 10 (*)	Bradley (1972)
Jamshid al-Kashi, calculating genius	5 – 10	Van Brummelen (1998)
Short history of mensuration	< 5	Dodd (1977)
Egyptian mathematics	< 5	Dubbey (1975)
Babylonian mathematics	< 5	Dubbey (1976)
Mathematicians until 400 AD	< 5	Duparc (1970)
Legends about Pythagoras	< 5	Fant (1969)
Kaleidoscope	> 10	Graf/Hodgson (1990)
Archimedes' volume of a sphere	5 – 10	Del Grande (1993)
Quadrature of the circle	5 – 10	Hallerberg (1978)
Descriptive geometry	> 10 (*)	Den Hartog (1981)
Approximation of π	5 – 10	Hogendijk (1980)
Approximations of the direction of Mekka	> 10 (*)	Hogendijk (1993)
Theorem of Ceva	< 5	Lightner (1975)
Greek mathematical diagrams	5 – 10	Netz (1998)
Short history of trigonometry	> 10	Norman (1977)
Conic curves	> 10	Parr (1977)
The work of Evangelista Torricelli	< 5 (*)	Robinson (1994)
Non-Euclidean geometry	5 – 10	Rolwing/Levine (1969)
Women in the history of mathematics	5 – 10	Rothman (1997)
Roman geometry	5 – 10	Röttel (1981)
Golden ratio and π	< 5	Seitz (1986)
The work of Ibn al-Haytham (Alhazen), 11th C.	> 10	Smith (1992)
Regular polyhedra	> 10	Stengel (1972)
Ancient Chinese mathematical work: the *Chiu-chang suan-shu*	> 10	Swetz (1972)
Geometrical structure of the sriyantra	5 – 10	Tahta (1992a)
Geometrical exploration of a painting by Piero della Francesca	5 – 10	Tahta (1992b)
Constructions with compasses and ruler	< 5	Waters (1982)
Ptolemeus' problem	< 5 (*)	Weeks (1998)

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